# On momentum dependence of the reaction $\pi^-p \to \omega n$ near threshold

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**Abstract.** We discuss the near–threshold behavior of the $\omega$ production amplitude in the reaction $\pi^-p \to \omega n$. In contrast to the results of earlier analyses we find that the averaged squared matrix element of the production amplitude must be a decreasing function of energy in order to describe the existing experimental data.

**PACS.** 13.60.Le Meson production – 25.80.-e Meson- and hyperon-induced reactions

## 1 Introduction

The reaction $\pi^-p \to \omega n$ near threshold was studied relatively long ago [1–3]. The authors of those papers claimed to have found an abnormal behavior of the production amplitude for this reaction near threshold that is not yet understood theoretically [4]. This conclusion was based on a comparison of the measured cross section with that for the production of a stable particle. Specifically, they found that the production cross section is proportional to $P^{*\,2}$ instead of $P^*$, as expected. ($P^*$ denotes the momentum of the outgoing neutron in the center of momentum system.) This behavior was interpreted as possible evidence for a resonance in the $\omega N$ system not far above threshold [2]. At the same time there are no direct indications of the existence of such a resonance in the $\pi^-p$–channel. Recently a behavior similar to that of the cross section under discussion was also found in the reaction $pd \to \omega\,^3He$ [5].

The transition amplitude $\pi N \to \omega N$, while interesting in its own right, is also of great importance in other reactions. Theoretical analyses of the reactions $pp \to pp\omega$ [4], $pn \to d\omega$ [6] and $dp \to\,^3He\omega$ [7], as well as $\omega$ production in proton–nucleus collisions [8] all rely on the $\pi N \to \omega N$ transition amplitude, in which the pion enters as an exchanged particle, as the basic mechanism for the reactions studied. It is thus of great theoretical interest to obtain direct, reliable experimental information on this reaction.

The experiments cited above were all performed in an unusual kinematical situation: instead of measuring the momentum distribution of the final state for a fixed beam energy, the excitation function for a fixed neutron momentum versus initial energy was measured.

In this work we analyze the general expression for the production cross section of unstable particles in near–threshold binary reactions. Both situations – the standard one, wherein the energy is fixed and the final state momentum is varied – and that of the above experiments, wherein one of the final momenta is fixed while varying the energy, are compared. We shall demonstrate that the dependence of the count rates on the outgoing center of mass momentum depends on how the analysis is done. We conclude that the behavior of the $\omega n$ amplitude is quite normal. That means that the earlier interpretation of the experimental data [1–3,5] is incorrect. To explain the results of the cited papers, we need a smooth behavior of the averaged matrix element that must be a decreasing function of energy in the near–threshold region.

We begin by discussing the production of a resonance using a monochromatic beam. We then consider the integration of the so-obtained cross section over the beam energy, which is the procedure that was carried out experimentally in [1–3]. We close with a discussion of the formulae used in the cited papers to analyze the experimental data.

## 2 Cross section for the production of an unstable particle

Let us consider the case of a monochromatic beam of energy $E$[1]. In this case the differential cross section $d\sigma/d\Omega$

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[1] Note, that all kinematical quantities are given in the center of mass.

for production of a stable particle is

$$
\frac{d\sigma}{d\Omega}\bigg|_{stable} = \frac{\mu_i (2\pi)^4}{p_i}
$$
$$
\times \int |T(E,\boldsymbol{k})|^2
$$
$$
\times \delta(E - M - m - k^2/2\mu)k^2 dk, \quad (1)
$$

where $\mu_i$ ($\mu$) and $p_i$ ($k$) denote the reduced mass and the momentum of the initial state (final state) respectively. This integral is proportional to $k(E,m) = \sqrt{2\mu(E - M - m)}$ after the integration is performed. If we consider the cross section for the production of an omega meson or any other resonance with finite width $\Gamma$, expression (1) must be convoluted with the spectral density $\rho(m, \Gamma)$. For simplicity we use a Breit-Wigner form for the spectral density, namely

$$
\rho(m, \Gamma) = \frac{\Gamma/2\pi}{(m - \bar{m})^2 + \Gamma^2/4} . \quad (2)
$$

Here $\bar{m}$ is the average mass of the unstable particle. In this case the resonance production cross section is given by the expression:

$$
\frac{d\sigma}{d\Omega}\bigg|_{unstable} = \frac{\mu_i (2\pi)^4}{p_i}
$$
$$
\times \int_0^{K_{max}} \frac{\Gamma/2\pi}{(E_{kin} - k^2/2\mu)^2 + \Gamma^2/4}
$$
$$
\times |T(E,\boldsymbol{k})|^2 \, k^2 dk , \quad (3)
$$

where $E_{kin} = E - M - \bar{m}$ and $K_{max}$ is the maximum momentum of the outgoing neutron for the reaction $\pi^- p \to \omega n$. $K_{max}$ is determined by the masses of the lightest decay products of the unstable particle.

Note that, because of the experimental setup, the authors of the papers [1–3] have not measured the total differential cross section for the production of an unstable particle as given by (3), but only a fraction of it, as the momentum of the outgoing neutron was constrained to lie in a small band around a given $P^*$. In other words, they have measured the following part of differential cross section:

$$
\frac{d\sigma}{d\Omega}\bigg|_{\Delta P} = \frac{\mu_i (2\pi)^4}{p_i}
$$
$$
\times \int_{P^*-\Delta P/2}^{P^*+\Delta P/2} \frac{\Gamma/2\pi}{(E_{kin} - k^2/2\mu)^2 + \Gamma^2/4}
$$
$$
\times |T(E,\boldsymbol{k})|^2 \, k^2 dk , \quad (4)
$$

Let us estimate the remaining integral for the case when the scattering amplitude $|T(E,\boldsymbol{k})|$ is approximately constant in the interval $\Delta P$, as one would expect close to the production threshold. In this case we get

$$
\frac{d\sigma}{d\Omega}\bigg|_{\Delta P} \propto |T(E, P^*)|^2 I(E_{kin}), \quad (5)
$$

where

$$
I(E_{kin}) = \frac{\mu}{\pi}\sqrt{\mu\Gamma} \int_{a_-}^{a_+} \frac{\sqrt{y}\,dy}{(y - y_0)^2 + 1} \quad (6)
$$

and $y_0 = 2E_{kin}/\Gamma$. Here the limits are $a_\pm = \frac{(P^* \pm \Delta P/2)^2}{\mu \Gamma}$. As will become clear below, the behavior of the integral depends on the parameter

$$
\chi(P^*) := a_+ - a_- \equiv \frac{2P^* \Delta P}{\mu \Gamma} . \quad (7)
$$

Let us consider the case of small $P^*$ and $\Delta P$ such that the condition

$$
I(E_{kin}) \approx \chi(P^*) \ll 1 \quad (8)
$$

is satisfied. In this limit the denominator under the integral is practically constant, so that we have

$$
I(E_{kin}) \approx \frac{\mu}{\pi}\sqrt{\mu\Gamma}\frac{1}{(y^* - y_0)^2 + 1} \int_{a_-}^{a_+} \sqrt{y}\,dy
$$
$$
= \frac{2}{\pi\Gamma} \frac{P^{*2}\Delta P + \frac{1}{12}(\Delta P)^3}{(y^* - y_0)^2 + 1} , \quad (9)
$$

where $y^* = P^{*2}/\mu\Gamma$. The dependence of this integral on energy looks like a BW-resonance with strength proportional to $P^{*2}$. This was the dependence found in [1].

In the experiments [1–3] an additional integration over the beam energy (still keeping $P^*$ fixed) was performed in order to remove the width-dependence from (9). Indeed, since the spectral density is normalized, integrating over the beam energy gives

$$
\int dE_{kin} I(E_{kin}) \simeq P^{*2}\Delta P + \frac{1}{12}(\Delta P)^3 . \quad (10)
$$

The above derivation shows specifically that

$$
|T(E, P^*)|^2 \propto \frac{1}{g(P^*)}\frac{d\sigma}{d\Omega}\bigg|_{\Delta P} , \quad (11)
$$

where $g(P^*) = P^{*2} + \frac{1}{12}(\Delta P)^2$. The right hand side of this equation is displayed in Fig. 1, based on the data of [3] for the points with $P^* \geq 50 MeV/c$. As for the point at $30 MeV/c$, we used the data from [2], averaged over an interval of $\Delta P = 20 MeV/c$. Note that the errors of $(d\sigma/P^{*2})$ displayed in the plot contain the uncertainty in $P^*$ as well. Figure 1 gives evidence for a matrix element $|T(E, P^*)|$ that is practically constant, at least for the points $P^* \leq 110 MeV/c$. The data above $110 MeV/c$ give evidence for a matrix element that decreases smoothly with $P^*$, as would be expected in the usual effective range approximation. We conclude therefore that the existing experimental data [1–3] for the reaction $\pi^- p \to \omega n$ give no indication of a growth of the matrix element for increasing $P^*$ in a wide interval of momenta $P^*$ above threshold. Note that the over–all dependence of the matrix element on $P^*$ is contrary to the conclusions of [1-3].
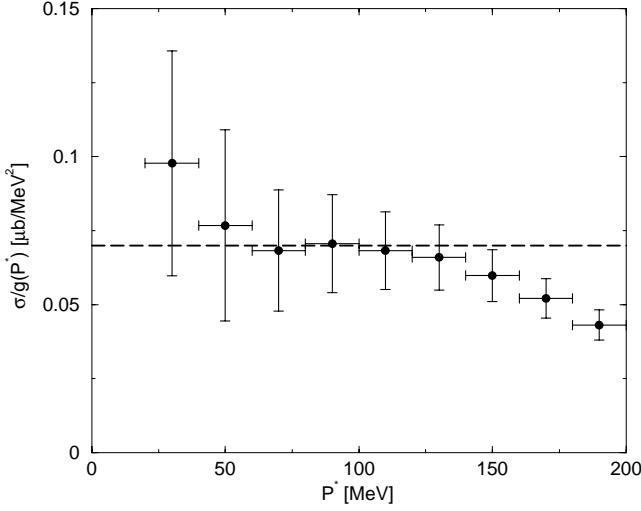
**Fig. 1.** The cross section data normalized to $g(P^*) = P^{*2} + \frac{1}{12}(\Delta P)^2$. The curve is introduced to guide the eye. The data are from [2,3], where the errors where modified according to the uncertainty in $P^*$

We now investigate the second limiting case,

$$\chi(P^*) \gg 1. \qquad (12)$$

To estimate the integral (6) in this situation we must further distinguish separately two possibilities:

i) The energy parameter $y_0$ is within the limits $a_+$ and $a_-$ of the integral (6). In this case

$$I(E_{kin}) \approx \sqrt{2\mu E_{kin}}. \qquad (13)$$

ii) The energy parameter $y_0$ is not within the interval $[a_-, a_+]$. The integral $I(E_{kin})$ is then strongly suppressed.

In short, if condition (12) is satisfied we get the usual energy behavior for the differential cross section, namely a linear $P^*$–dependence.

The condition

$$\chi(P^*) \approx 1$$

determines the critical value of $P^*$. Thus, by measuring the count rates versus $P^*$ one may observe a transition from a $P^{*2}$ behavior of the cross section at low $P^*$ to a linear dependence at high $P^*$, even for a constant matrix element. In the case of the omega this takes place at $P^*_{cr} = \mu\Gamma/2\Delta P \approx 90 MeV/c$, if $\Delta P \approx 20$ MeV, as specified in [2].

In [1] the production of $\eta$ and $\eta\prime$ was studied as well. The authors report that here a behavior very different from what they found for $\omega$ production. Using the above discussion one can now easily understand this: for both mesons condition (12) was satisfied, since $P^*_{cr} = 0.01$ MeV for the $\eta$ and $P^*_{cr} = 2.4$ MeV for the $\eta\prime$.

To complete our criticism of the analyses of [1–3], we compare our formulae to those given therein. Let us start by briefly repeating the arguments for a linear dependence of the $\omega$ production cross section on $P^*$ given in
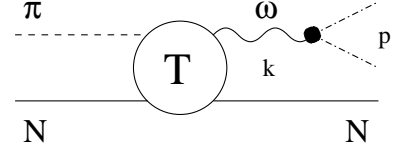


**Fig. 2.** Illustration of the reaction $\pi N \to \omega N$ as a three particle reaction taking into account the decay of the $\omega$

[1]. Instead of (1) for the production of a stable particle, [1] starts directly from the expression for the double–differential cross section,

$$\frac{d^2\sigma}{dm\,d\Omega} \propto \rho(m, \Gamma)P^* . \qquad (14)$$

In order to get the total production rates for producing final particles with a given $P^*$, expression (14) was integrated over the initial energy under the constraint $P^* = const$. By employing energy conservation, i.e. using the condition

$$dm = dE \quad \text{for} \quad P^* \text{ fixed} , \qquad (15)$$

(14) can be formally integrated. Since the spectral density is normalized, this integration yields

$$\frac{d\sigma(P^*)}{d\Omega} \propto P^* . \qquad (16)$$

This procedure to obtain (16) from (14) looks formally correct, but it is not. The reason for this is that in order to derive (14) the energy conserving $\delta$–function in (1) was evaluated. Therefore $P^*$ in (14) implicitly depends on $E$ and $m$ and thus must be treated as a dependent variable in any argument based on (14). Therefore the use of the relation (15) in this context is simply incorrect. Instead, the condition of allowing $P^*$ to vary only in a small interval translates into a condition on the ranges of integration of $m$, given a fixed energy $E$, namely

$$\frac{d\sigma}{d\Omega} \propto \int_{m_0 - \chi\Gamma/2}^{m_0 + \chi\Gamma/2} \rho(m, \Gamma)P^* , \qquad (17)$$

with $m_0 = E - M - P^{*2}/2\mu$. This formula actually agrees with our (5) if we rewrite it in terms of an integration over $dm$. Therefore we conclude that in the theoretical analysis of [1–3] the limits of integration were not properly treated, thereby leading to an inappropriate conclusion for the momentum dependence of the cross section.

It is this point that was overlooked in the earlier works. If we impose the limit $\Delta P \to 0$ on (17) and integrate over the energy we again find

$$\frac{d\sigma}{d\Omega} \propto P^{*2}\Delta P . $$

To clarify the situation we would like to add that the final result agrees with what one expects when taking the decay of the unstable particle into account explicitly. Let

us, for simplicity, assume a two particle decay[2], as illustrated in Fig. 2. In this case the phase space is the 3 body phase space and we get

$$d\sigma \propto d^3k \, d\Omega_p \mu_d |\boldsymbol{p}| \mid T(E, \boldsymbol{k}) D_\omega(P^2) W(m,p) \mid^2 \, , \quad (18)$$

where $D_\omega$ denotes the dressed $\omega$ propagator, $W$ is the decay amplitude, $\mu_d$ is the reduced mass of the decay particles and $p$ their relative momentum. Therefore (18) agrees with (3) when we identify

$$\rho(m, \Gamma) = \int d\Omega_p \mu_d |\boldsymbol{p}| \mid D_\omega(m) W(m,p) \mid^2$$
$$= -\frac{1}{\pi} Im(D_\omega(m)), \quad (19)$$

where we used unitarity for the second identity. Equation (19) agrees with the standard definition of a spectral function.

## 3 Summary

To summarize, we demonstrated that the interpretation of the experimental results for the reaction $\pi^- p \to \omega n$ given in [1–3] is incorrect. A proper treatment of the independent variables leads to an expression for the momentum dependence of the integrated cross section that is consistent with a constant matrix element near the production threshold.

The procedure of [1–3] was also used in [5] and thus our criticism applies to the conclusion of this paper as well. However, we want to emphasize that we regard the method of integrating over the beam energy while keeping the final momentum fixed as useful way to examine the production of narrow resonances close to their production threshold. This technique allows for a more direct access to the production amplitude and, simultaneously, to an increase in the counting rate.

We demonstrated that as the momentum $P^*$ grows, the formula for the cross section reduces to the standard one for the production of a stable particle, as expected. The relevant parameter is $\chi = \frac{2P^* \Delta P}{\mu \Gamma}$.

The knowledge of the transition amplitude $\pi N \to \omega N$ is an important input for several approaches investigating $\omega$ production in hadron–hadron collisions [4,6–8]. A better understanding of its energy dependence therefore will help us to get deeper insight in the strong interaction of vector mesons and nucleons and nuclei in the intermediate energy regime.

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[2] What follows is exact under the assumption that the unstable particle decays into this channel only. However, the generalization is straightforward and only complicates the argument.